The Indian Rope Trick: Rope vs Compass-Box

C. K. Raju
Centre for Studies in Civilizations
36 Tughlakabad Institutional Area
New Delhi 110 067
c_k_raju@vsnl.net

The development of the "infinitesimal" calculus in India was facilitated by the use of a flexible rope as the basic geometrical instrument—used to measure the length of a curved line. This made manifest the in-principle meaning of the length of a curved line—something declared to be beyond the capacity of the human mind by Descartes who based himself on the straight line. The compass-box reinforces this difficulty by suggesting the straight line as the basis of geometry. Since the rope can perform the function of each instrument in the compass-box, the low-cost rope (or string) is suggested as a superior replacement for the currently-used compass-box.

Introduction

Some recent publications (Raju, 2001, 2007) have brought out the historical development of the "infinitesimal" calculus in India, over a thousand years, and its subsequent transmission to Europe where it was attributed to Newton and Leibniz by Western historians of science who have systematically denied non-Western contributions to science.

Some of the cultural factors that facilitated the invention of the calculus in India were specific to India. For example, Indian wealth depended on agriculture, which depended on the monsoons, necessitating a precise calendar to mark the ("erratic") rainy season. The calendar was made for a central meridian (of Ujjaini) and recalibrated across a large cultural area, India, by determining local latitude and longitude, and the size of the globe, using trigonometry. Precise trigonometric values were required for this. Geometrical techniques being cumbersome, precise trigonometric values were obtained in the 5th c. CE using an elegant finite difference technique which developed into the calculus over the next thousand years, as the demand for precision increased.

Other cultural factors were more general, and the aim of this note is to point out the potential value, for current school education, of one such cultural factor.

The calculus began with the determination of the length of the circumference of a circle in units of the radius, or the calculation of the number today called π.[1] How does one determine the length of a curved line?

The compass-box

This question is not easy to answer today just because the compass-box is so essential an aspect of a child's school kit. The box strongly suggests the straight line as the basic notion of geometry.

The geometry presupposed by the compass-box is metric rather than synthetic[2] for the box has a scale (rather than an unmarked straight edge) and compasses (which are not "collapsible"), so that distances *can* be picked and carried, and it is meaningful to measure lengths. However, the rigid scale allows one to measure the length of only straight line segments. The compass-box does not provide any instrument to measure the length of a curved line segment. One cannot measure even the length of an arc of a circle, only the angles in degrees that various circular arcs might form—hence many children never understand the natural radian measure, which depends upon being able to measure the length of the arc. Hence, also, most students are more comfortable with 360° than with 2π.

In fact, students acquire only an operational understanding of the notion of a degree by using the protractor. Since an angle is defined not as the relative length of a circular arc, but as something connected with pairs of straight line segments, students remain woolly about the meaning of a degree. A degree is the 90th part of a right angle, but what is the entity which is being divided into 90 equal parts? The *natural* way to answer this question is to point to the circular arc, but that presupposes a way to measure its length.

It would not be accurate to say that the compass-box is based on "Euclidean" geometry, since metric geometry trivializes the *Elements* (while Hilbert's synthetic interpretation does not (Raju, 2001b) fit the entire *Elements*). Nevertheless, one might say that the present-day compass-box still suffers from a hangover of idealism (Raju, 2006) ("Platonism", "Neoplatonism") in regarding the straight line as the

[1] The first determination of π is often chauvinistically attributed to Archimedes. But the earliest evidence for this comes from "translated" manuscripts some 1600 years after Archimedes, which cannot credibly be connected to Archimedes or even to his 6th c. CE commentator Eutocius.

[2] For more details on synthetic geometry, see Moise (1963); for a comparative account of the instruments used by various types of geometry, see Raju (2001a).

ideal figure underlying geometry.

This hangover presumably relates also to the practical value of navigation, for the compass-box also mimics the tools of the European navigator who, proceeding from an idealistic understanding of geometry, took the straight line as the primary geometric figure. The dependence of European navigators on the straight line became evident in the 16th c. CE when they started making long voyages across the sea. Over short distances, such as those in the Mediterranean sea, the surface of the sea could be regarded as approximately plane. On a plane surface a ship steering a constant course (set by, say, a magnetic compass or by the straight line joining two stars) should trace a straight line. However, on the globe, the ship traces a *curved* line, called a loxodrome (from *loxos* = oblique, and *dromos* = curve). Except in cardinal directions, this curve is not even a great circle as European navigational theorists like Nunes initially took it to be. Because European navigational techniques were so dependent upon the straight line, European navigators in the 16th c. CE could not navigate without charts which showed loxodromes[3] as straight lines. Hence, the great value of the Mercator chart (common "map of the world") in which loxodromes are straight lines. So great was the value of this chart to European navigators that subsequent British naval supremacy is put down to a better understanding of this chart! Because the compass-box mimics the European navigator's paraphernalia, though set squares and dividers are rarely used, they ritually remain part of the box.

Indian rope geometry

Now, India has had an old tradition of geometry from the days of the *śulba sūtra*-s (-6th c. CE), which precede Greek geometry. The "śulba" refers to a rope, and "rajju", also meaning rope[4], or string, was a common part of the Indian school syllabus in pre-British times. It is still used by artisans, but is no longer taught in formal schools—such practical things are looked down upon from the ("Platonic") point of view of geometry regarded as high metaphysical discourse.

The introduction of British education in India, even in contexts where there was no noticeable colonial plot, sometimes made local people conceptually and technologically dependent on remote foreign sources, while teaching techniques that were inferior to local techniques. Let us see this in the case of the rope vs the compass-box.

The rope can be used to do a number of things.

1. By holding it taut one can draw a straight line, so it can perform the function of a straight edge.

2. By choosing any appropriate unit, it can be made into a scale (as in a measuring tape). Traditionally, knots were used for marks. This "primitive" technique, when combined with the two-scale principle (nowadays called the Vernier principle) as in early navigational instruments like the *kamāl*, gives a remarkably high degree of accuracy. (In particular, the *kamāl* I obtained from the Lakshadweep islands, though ridiculed as "primitive" by 19th c. British historians, could measure angles accurate to $10'$ of the arc or $\frac{1}{6}^\circ$. British school education replaced it by a steel sextant, and the sextant the islanders could afford was less accurate being graduated in degrees.[5])

3. By keeping one end fastened and moving the other end around, one can draw a circle. So the rope performs the function of a compass.

4. Most importantly, a rope can be used to directly measure the length of the arc, hence an angle in radians: simply lay the rope along the curve, and straighten it to compare with the length of a straight line segment.[6] Because it can be used to measure circular arcs, a rope also serves as a protractor which measures angles in radians.

5. By marking two points on it, a distance can be picked and carried, so a rope (or string) can perform the function of a divider.

6. Using the "fish figure" (the figure in *Elements* 1.1), it is easy to construct a right angle, and by bisecting or trisecting it, it is easy to construct angles of 45°, 30°, and 60°, so a rope is also a substitute for set squares.

7. By fastening two points, one can also draw an ellipse with the rope. This is impossible with the compass-box.

So, a rope (or a piece of twine) can be used to do everything that can be done with a compass-box, and something more: it can measure the length of both straight and curved lines. The most important new capability is the fourth one, above, for it directly assigns a meaning to the length of the arc, or the length of a curved line. This is the sort of meaning that a child can easily grasp (with the hand as well as the mind). The same thing is not possible with a ruler, and assigning a meaning to a curved line starting from straight lines requires the calculus.

[3] Since the calculation of loxodromes involves the determination of a curve given its tangent at every point, this is equivalent to the fundamental theorem of calculus (Struik, 1969), although the solution to this problem was known in Europe about a century before the calculus officially arrived there.

[4] This is the traditional translation, since the English language does not have the diminutive for rope that is meant, so that the "rope" in question is understood to be thin. "Rope" does not quite mean "string" which might be extensible, but is closer to "twine", although the fibre used could be jute rather than cotton. Since this article concerns mathematics rather than rope-technology, one could very well substitute "rope" by a flexible measuring tape, as appropriate.

[5] The *kamāl* was the instrument used by the Indian navigator, Malmi Kanha, who brought Vasco da Gama from Melinde to India across the "uncharted" sea which Vasco da Gama could not possibly have navigated on his own, for, like Columbus, he knew no celestial navigation. The two-scale principle used in the *kamāl* is elaborated in Raju (2007) chp. 6. The *kamāl* is more sophisticated since it does harmonic interpolation rather than linear interpolation. The use of the two-scale principle for harmonic interpolation was not explicit in earlier publications on the *kamāl*, including the one by this author.

[6] A measuring tape could be laid "edge-on" along the curve

The difficulty

As already stated, students accustomed to the compass-box find it difficult to grasp the notion of the length of a curved line. The level of difficulty involved is made clearer by the reaction of a leading Western thinker, regarded as one of the founders of modern geometry, René Descartes, when he was first exposed to the notion of the length of a curved line. Descartes went so far as to assert that assigning a meaning to the length of a curved line, using straight lines, was beyond the capacity of the human mind!

> the ratios between straight and curved lines are not known, and I believe cannot be discovered by human minds, and therefore no conclusion based upon such ratios can be accepted as rigorous and exact.(Descartes, 1996, Book 2, p. 544.)

Descartes' reaction was not an idiosyncratic one. Another great name of the times was Galileo, who privately raised similar objections about the calculus in his letters (Mancosu, 1996) to Cavalieri—hence Galileo allowed Cavalieri to publish on the calculus, but did not publish on it himself for he was unwilling to jeopardize his reputation.

The basic difficulty noticed by Descartes, Galileo et al. is that to measure a curved line, using straight lines, one requires an infinity of infinitesimal line segments, and these thinkers thought that the concept of an infinity of infinitesimals brought in problems of the sort that were best left to the divine. (The alternative was to work with an approximation, and approximations, according to these thinkers, were no part of mathematics which they regarded as exact.) Berkeley's devastating critique of Newton and Leibniz, articulated a century later (Berkeley, 1734), proceeded on similar grounds, and could simply not be answered by his contemporaries (Jurin, 1735; Robins, 1735).

The genesis of these epistemological and pedagogical difficulties is, in a way, captured by the differences between rope geometry and compass-box geometry: historically speaking, the notion of the length of a curved line followed the arrival of the calculus in Europe, while the notion preceded the development of the calculus in India—just because with a rope there is nothing mysterious about the length of a curved line.

On the principle that phylogeny is ontogeny, one can expect the difficulties (raised e.g. by Descartes, Galileo and Berkeley) about the length of curved lines to be repeated innumerable times in the minds of students as history repeats in the classroom today. Today, a satisfactory answer to those difficulties is believed to have been found with the formalisation of real numbers, and the formalisation of set theory needed for that. However, it took a couple of centuries to arrive at this formalist answer, which is too complex to be taught at the K-12 level today, where one still wants to teach the calculus for its practical value. Replicating the historical process of development specific to Europe, courses on calculus are today separated from courses on analysis. The calculus is taught for its practical value, without adequate understanding, while courses on analysis teach real numbers and limits (but still assume naive set theory).[7] From the perspective of ontogeny as phylogeny, using a rope instead of compass box would correspond to taking into account the entire historical development of the calculus, and not merely the developments specific to the West.

Other considerations

Apart from the epistemological angle, we can also consider the situation from the economic angle, which is important if we want to take education to poorer people in India. In India, a person is defined as poor if that person lacks enough income to regularly purchase two square meals (2400 Calories) a day. In this context, a compass-box or geometry set is expensive. It uses metals and plastics, and cannot be built locally. Most poorer children who cannot afford to purchase books do not purchase compass-boxes. If they do, and a piece goes missing it is not replaced. The compass-box is designed for use with pencil and paper (and sharpener and eraser) all of which add to the "running costs". These costs might be trifling in the US, but they are non-trivial in India, and unaffordable for a large group of poor students for whom the free mid-day meal offered in schools is a major attraction.

Finally, we can also look at the ecological angle. A rope or string can be used to draw figures even on the ground, although one *could* easily design it for use with pencil and paper if one wanted to, and also add markings, as in a measuring tape, to make it function like a scale. This re-usability (of the ground) with the string also makes it more eco-friendly than even acid free paper! Ironically, this is also appropriate to the conditions of many Indian schools, since, over the last half a century, the Indian government has consistently managed to provide ample luxuries for government officials but has not been able to provide classrooms for poor villagers, even though the Indian Constitution guarantees free education for all children, but does not guarantee luxuries for government officials.

Conclusions

The rope (or string) is flexible in more ways than one and can be used to do everything that can be done with a compass-box. It can further be used to measure the length of a curved line, impossible with the instruments in a compass-box. This is helpful for the measurement of angles, and the subsequent transition to trigonometry and calculus. The rope is also inexpensive, locally-constructible, eco-friendly, and suited to conditions prevalent in countries like India. Hence, it is a superior replacement for the compass-box.

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[7] For more on the pedagogy, see Raju (2005).

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